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Nusselt numbers in laminar flow for H2 boundary conditions

M. SPIGA and G. L. MORINI

Department of Energy, Nuclear and Environment Engineering, Facoltà di Ingegneria, University of Bologna, Viale Risorgimento 2, 40136 Bologna, Italy

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Abstract—The present analysis deals with the velocity and temperature distribution in steady state, hydrodynamically and thermally fully developed, laminar forced flow for Newtonian fluids in rectangular ducts. The analytical solutions are presented for all the eight modified H2 thermal boundary conditions around the periphery of the duct cross-section. The velocity profile represents the rigorous solution to the Navier–Stokes equation; the temperature profile is obtained resorting to series solutions of trigonometric functions. The temperatures and the Nusselt numbers are predicted as functions of the aspect ratio and compared with the results available in literature. Finally, very simple polynomial representations are given for the Nusselt numbers.

INTRODUCTION

The analysis of the hydrodynamics and thermal behavior of laminar flow through rectangular ducts is of special interest in a wide variety of heating and cooling devices used in compact or spiral heat exchangers, chemical reactors, meandering rivers, electronic cooling. Extensive numerical and experimental studies have been carried out by many authors, however the analytical investigation is generally disregarded. In fact the theoretical investigation, concerning the fluid temperature profile and the Nusselt number prediction, is very complicated because it requires a two- or three-dimensional (3D) analysis. Thermal boundary conditions in rectangular ducts are also complex, because there are many possibilities to impose different temperatures or heat fluxes on the four sides.

In order to accurately interpret the heat transfer problem, a clear understanding of the thermal boundary conditions is essential.

Recent reviews [1–3] have proposed a systematic exposition, restricted mainly to three classes of conditions: (1) T condition, i.e. constant temperature on the boundaries; (2) H1 conditions, i.e. constant wall temperature and constant fluid axial heat flux and (3) H2 condition, i.e. constant wall heat flux and constant fluid axial heat flux.

If the four sides of the rectangle have different temperatures or heat fluxes, the usual nomenclature must be modified. In a recent paper Gao and Hartnett [4] have focused the lack of analytical solutions for the H2 condition, and presented an analytical study for laminar slug flow in rectangular ducts, for eight versions involving different combinations of heated and adiabatic walls, symbolically specified as follows:

4: four heated sides;
3L: three heated sides and one adiabatic short side;
3S: three heated sides and one adiabatic long side;
2L: two heated long sides and two adiabatic short sides;
2S: two heated short sides and two adiabatic long sides;
2C: one short and one long heated sides (corner version);
1L: one heated long side and
1S: one heated short side.

The solutions for slug flow in [4] cannot be applied to Newtonian flows, where the spatial effects of the velocity distribution are very effective, mainly near the corners and along the narrow walls. Hence it is essential to consider the actual velocity profile in the fluid as a function of two Cartesian coordinates (in fully developed flow). Aim of this paper is the rigorous determination of the temperature profile of a Newtonian fluid, in laminar, hydrodynamically developed flow, through rectangular ducts, with constant different heat fluxes on the four sides of the wetted perimeter. Then the Nusselt number can be easily predicted by integrating simple temperature differences along the wetted perimeter of the rectangular cross-section.

STATEMENT AND SOLUTION

Consider a Newtonian fluid, in incompressible laminar hydrodynamically fully developed flow, in forced convection, in a rectangular duct with axially unchanging cross-section.

In order to solve the energy equation let us assume that a constant heat flux q is exchanged between the fluid and the wall, along a rectangular perimeter of arbitrary heated length L; an energy balance between

NOMENCLATURE

A	parameter defined in equation (3)
a, b	longer and shorter sides, respectively, of the rectangle [m]
B_{mn}	coefficients defined in equations (9)
c^*	function of the combination of heated and adiabatic walls
d_1, d_2, d_3, d_4	constants defined in Table 1
D	hydraulic diameter of the duct $2ab/a+b$ [m]
g_i	ith polynomial coefficient in Nusselt number representation
h	heat transfer coefficient [W m^{-2} K^{-1}]
j, k, m, n	summation indices
K	fluid thermal conductivity [W m^{-1} K^{-1}]
L	heated perimeter length on the rectangular cross section [m]
Nu	Nusselt number for laminar flow, hD/K
Nu^*	Nusselt number for slug flow
q	heat flux through the duct walls [W m^{-2}]
t_{max}	dimensionless maximum wall temperature defined in equation (13)
t_{min}	dimensionless minimum wall temperature defined in equation (14)
$T(\cdot)$	fluid temperature [K]
$v(\cdot)$	axial velocity for fully developed laminar flow [m s^{-1}]
W	fluid average velocity [m s^{-1}]
x, y, z	dimensionless rectangular Cartesian coordinates.

Greek symbols

α	fluid thermal diffusivity [m^2 s^{-1}]
β	aspect ratio $b/a \leqslant 1$
ε	relative error between the polynomial representation and the exact solution
$\theta(\cdot)$	dimensionless fluid temperature
θ_c	dimensionless fluid temperature at the duct center
θ_{bulk}	dimensionless bulk temperature
θ_w	dimensionless mean wall temperature
$\theta_{w,max}$	dimensionless maximum wall temperature
$\theta_{w,min}$	dimensionless minimum wall temperature
$\tau(\cdot)$	dimensionless fluid temperature $\theta(\cdot)-B_{00}$
ξ, η, ζ	Cartesian coordinates [m].

the sections ζ and $\zeta+d\zeta$ gives the axial variation of the fluid temperature:

$$\frac{\partial\theta}{\partial z}=\frac{\alpha L(1+\beta)}{2Wb^2} \tag{1}$$

where the dimensionless temperature and the axial coordinate are $\theta = KT/qD$ and $z = \zeta/a$, respectively. Hence the fluid temperature presents a linear variation along the axial duct coordinate.

The origin of the Cartesian coordinate system is placed at the bottom left corner of the rectangle, and the fluid flow is directed toward the ζ axis; the ξ axis and the longer side of the rectangle are parallel. Under assumption of constant fluid properties, neglecting axial thermal conduction, natural convection, viscous dissipation, internal energy sources, with rigid and non porous duct walls, the differential steady state energy equation may be written as:

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=\frac{av(x,y)}{\alpha}\frac{\partial\theta}{\partial z}. \tag{2}$$

The non-dimensional coordinates and temperature are $x = \xi/a$ (with $0 \leqslant x \leqslant 1$), $y = \eta/a$ (with $0 \leqslant y \leqslant \beta$, being $\beta = b/a \leqslant 1$).

The velocity distribution appearing in the equation (2) has been recently obtained as an analytical solution to the Navier–Stokes equation [5], resorting to a double Fourier sine transform. In hydrodynamically developed flow there is only one nonzero component v; it reads as:

$$v(x,y)=\frac{W}{A}\sum_{j=1,\text{odd}}^{\infty}\sum_{k=1,\text{odd}}^{\infty}\frac{\sin(k\pi x)\sin\left(j\pi\frac{y}{\beta}\right)}{jk(\beta^2k^2+j^2)} \tag{3}$$

where the number A depends on the aspect ratio:

$$A=\frac{4}{\pi^2}\sum_{j=1,\text{odd}}^{\infty}\sum_{k=1,\text{odd}}^{\infty}\frac{1}{j^2k^2(\beta^2k^2+j^2)}. \tag{4}$$

The fast convergence of the double series is guaranteed by the third and fourth power of the integer indices j and k in the denominator of equations (3) and (4). The number A can be approximated, with an agreement less than 0.1% with respect to the exact value of equation (4), by the third-order polynomial: $0.5059-0.3022\,\beta-0.0642\,\beta^2+0.0747\,\beta^3$.

The availability of a simple and symmetric solution for the velocity profile allows us to tackle the thermal problem by means of a rigorous analytical approach.

By substituting in the energy equation (2) the velocity profile stated in the equation (3) and the temperature partial derivative of equation (1), the following expression is obtained:

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=\frac{c^*}{A}\sum_{j=1,\mathrm{odd}}^{\infty}\sum_{k=1,\mathrm{odd}}^{\infty}\frac{\sin(k\pi x)\sin\left(j\pi\frac{y}{\beta}\right)}{kj(\beta^2k^2+j^2)} \tag{5}$$

where the number c^*, depending on the not yet specified heated length L, is obtained according to the energy balance and reads as:

$$c^*=\frac{L(1+\beta)}{2a\beta^2}. \tag{6}$$

The solution to the equation (5) is subject to the boundary conditions, that, according to the H2 approach, are:

$$\left(\frac{\partial\theta}{\partial x}\right)_{x=0}=d_1\left(\frac{1+\beta}{2\beta}\right)\quad\left(\frac{\partial\theta}{\partial x}\right)_{x=1}=d_2\left(\frac{1+\beta}{2\beta}\right)$$
$$\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=d_3\left(\frac{1+\beta}{2\beta}\right)\quad\left(\frac{\partial\theta}{\partial y}\right)_{y=\beta}=d_4\left(\frac{1+\beta}{2\beta}\right). \tag{7}$$

The constants d_1, d_2, d_3, d_4 and c^* depend on the specified combination of heated and adiabatic walls, imposed by the boundary conditions. The modified H2 thermal boundary conditions involve the heating of one or more of the four walls, with adiabatic remaining walls. With reference to the eight versions of these boundary conditions, the constants d_1, d_2, d_3, d_4 and c^* are shown in Table 1.

The H2 condition implies that the fluid temperature undergoes a linear variation along the duct length, hence the two-dimensional (2D) temperature distribution in the cross-section is simply shifted along the z axis.

Taking the velocity profile as a starting point for determining the solution to the equation (1), the unknown temperature distribution is sought by resorting to a double series in terms of complete systems of orthogonal functions:

$$\theta(x,y)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}B_{mn}\cos(n\pi x)\cos\left(m\pi\frac{y}{\beta}\right). \tag{8}$$

Then the problem is reduced to the determination of the constants B_{mn}. In order to obtain the solution, the first step consists of multiplying every term of equation (5) by $\cos(n\pi x)\cos(m\pi y/\beta)$ and by integrating over x, between 0 and 1, and over y, between 0 and β. The numerous integrals appearing in this procedure can be easily and patiently solved by classical methods. Many integrals disappear thanks to the orthogonality of the systems in equation (8); the boundary conditions directly appear when solving by parts the integrals involving the derivatives of temperature.

After some algebra, it is possible to obtain the unknown coefficients:
for $m\neq 0$ and $n\neq 0$:

$$B_{mn}=\begin{cases}-\dfrac{16c^*}{A\pi^4\left(\dfrac{m^2}{\beta^2}+n^2\right)}\displaystyle\sum_{k=1,\mathrm{odd}}^{\infty}\sum_{j=1,\mathrm{odd}}^{\infty}\\ \quad\times\dfrac{1}{(\beta^2k^2+j^2)(k^2-n^2)(j^2-m^2)} & \text{if } n \text{ and } m \text{ even}\\ 0 & \text{else}\end{cases} \tag{9a}$$

for $m=0$ and $n\neq 0$:

$$B_{0n}=\begin{cases}\dfrac{2}{n^2\pi^2}\left[(d_2-d_1)\left(\dfrac{1+\beta}{2\beta}\right)-\dfrac{4c^*}{A\pi^2}\displaystyle\sum_{k=1,\mathrm{odd}}^{\infty}\right.\\ \quad\left.\times\displaystyle\sum_{j=1,\mathrm{odd}}^{\infty}\dfrac{1}{j^2(\beta^2k^2+j^2)(k^2-n^2)}\right] & \text{if } n \text{ even}\\ -\dfrac{(d_1+d_2)(1+\beta)}{n^2\pi^2\beta} & \text{if } n \text{ odd}\end{cases} \tag{9b}$$

for $m\neq 0$ and $n=0$:

$$B_{m0}=\begin{cases}\dfrac{2\beta}{m^2\pi^2}\left[(d_4-d_3)\left(\dfrac{1+\beta}{2\beta}\right)-\dfrac{4\beta c^*}{A\pi^2}\displaystyle\sum_{k=1,\mathrm{odd}}^{\infty}\right.\\ \quad\left.\times\displaystyle\sum_{j=1,\mathrm{odd}}^{\infty}\dfrac{1}{k^2(\beta^2k^2+j^2)(j^2-m^2)}\right] & \text{if } m \text{ even}\\ -\dfrac{(d_3+d_4)(1+\beta)}{m^2\pi^2} & \text{if } m \text{ odd.}\end{cases} \tag{9c}$$

The coefficient B_{00} can not be determined, because

Table 1. Boundary condition for the eight versions of H2 boundary conditions

Version	d_1	d_2	d_3	d_4	c^*
1L	0	0	0	1	$\frac{(1+\beta)}{2\beta^2}$
1S	0	1	0	0	$\frac{(1+\beta)}{2\beta}$
2L	0	0	−1	1	$\frac{(1+\beta)}{\beta^2}$
2S	−1	1	0	0	$\frac{(1+\beta)}{\beta}$
2C	0	1	0	1	$\frac{(1+\beta)^2}{2\beta^2}$
3L	0	1	−1	1	$\frac{(1+\beta)(2+\beta)}{2\beta^2}$
3S	−1	1	0	1	$\frac{(1+\beta)(2\beta+1)}{2\beta^2}$
4	−1	1	−1	1	$\frac{(1+\beta)^2}{\beta^2}$

equation (2), worked out by the same procedure, with $m = n = 0$, provides the equality:

$$B_{00}[(d_2-d_1)\beta+(d_4-d_3)]\left(\frac{1+\beta}{2\beta}\right) = \beta c^* B_{00}. \quad (10)$$

In fact, for any version out of the eight versions quoted in Table 1, equation (10) is identically satisfied. The indetermination of B_{00} implies that the temperature distribution is calculated apart from an additive constant; this is physically consistent with the H2 conditions, because no temperature value is given either on the boundary or in the cross section. The Nusselt numbers depend only on temperature distribution, hence can be accurately and easily obtained. As usual [4], the Nusselt number related to the whole cross-section is:

$$Nu = \frac{1}{\theta_w - \theta_{bulk}} \quad (11)$$

where the dimensionless mean wall temperature is related to the heated walls only:

$$\theta_w = \frac{-d_1 \int_0^\beta \theta(0,y)\,dy + d_2 \int_0^\beta \theta(1,y)\,dy - d_3 \int_0^1 \theta(x,0)\,dx + d_4 \int_0^1 \theta(x,\beta)\,dx}{\beta(d_2-d_1)+(d_4-d_3)} \quad (12)$$

while the classical fluid bulk temperature is the double integral, over the rectangle, of the temperature $\theta(x, y)$, weighted on the velocity distribution indicated in equation (3). Specifying the constants d_1, d_2, d_3, d_4, the temperature profiles given in equation (8) and the Nusselt numbers are rigorously predicted for all the eight modified H2 boundary conditions. The thermal stresses occurring on the walls depend on the dimensionless maximum and minimum wall temperatures, defined by Shah in [6] for the H2 condition related to four heated walls:

$$t_{max} = \frac{\theta_{w,max} - \theta_c}{\theta_w - \theta_c} \quad (13)$$

$$t_{min} = \frac{\theta_{w,min} - \theta_c}{\theta_w - \theta_c} \quad (14)$$

where θ_c is the fluid temperature at the duct center. The temperatures t_{max} and t_{min} will be useful for comparison with some numerical results quoted in [6].

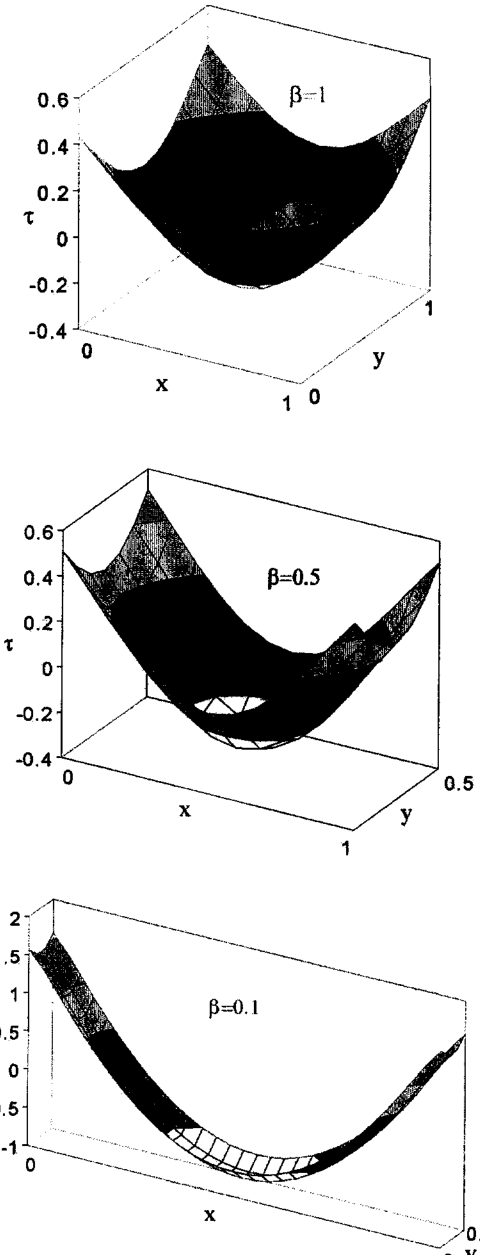

Fig. 1. Dimensionless temperature distribution, four heated walls.

RESULTS AND DISCUSSION

The previous results have been worked out on a modern PC equipped with 80486 processor. The availability of analytical expressions and the fast convergence of the double series in equation (8) make the present technique quite effective and inexpensive in terms of computer time. The 2D temperature distribution in the rectangle is reported, in Figs. 1–8, for different values of the aspect ratio, for all the eight combinations of heated and adiabatic walls. As previously pointed out, θ is defined apart from the constant B_{00}, hence the temperature $\tau(x, y) = \theta(x, y) - B_{00}$ is considered in the plots. The interpretation of the various temperature distributions, obtained by equation (8), is consistent with the physical perception.

In Fig. 1 the dimensionless fluid temperature distribution is sketched for the four heated walls (version

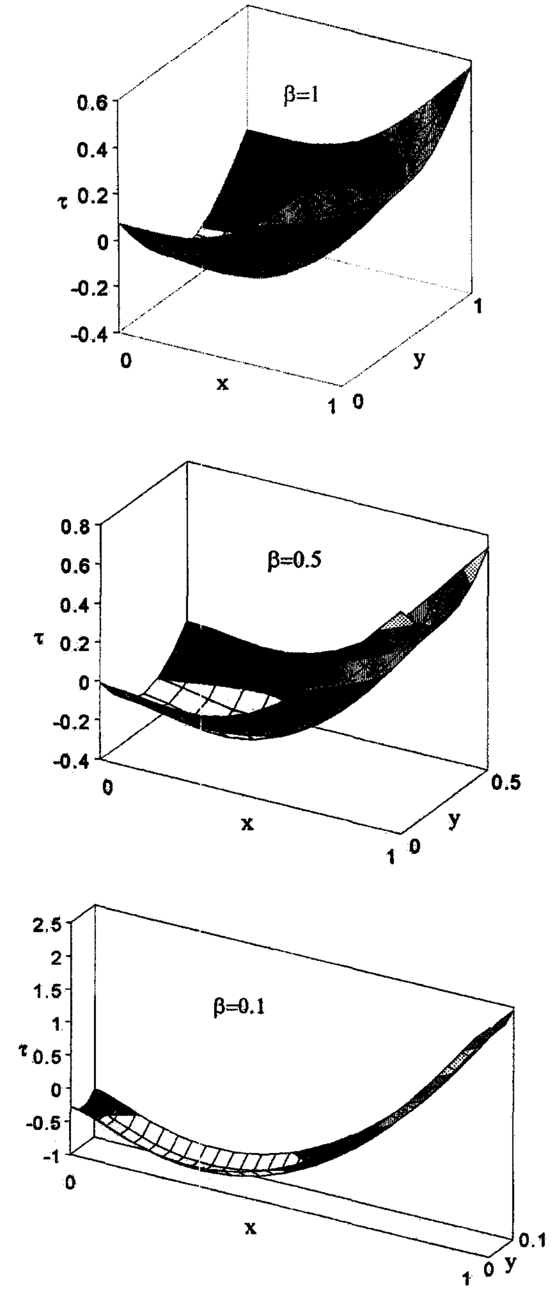

Fig. 2. Dimensionless temperature distribution, 3L version.

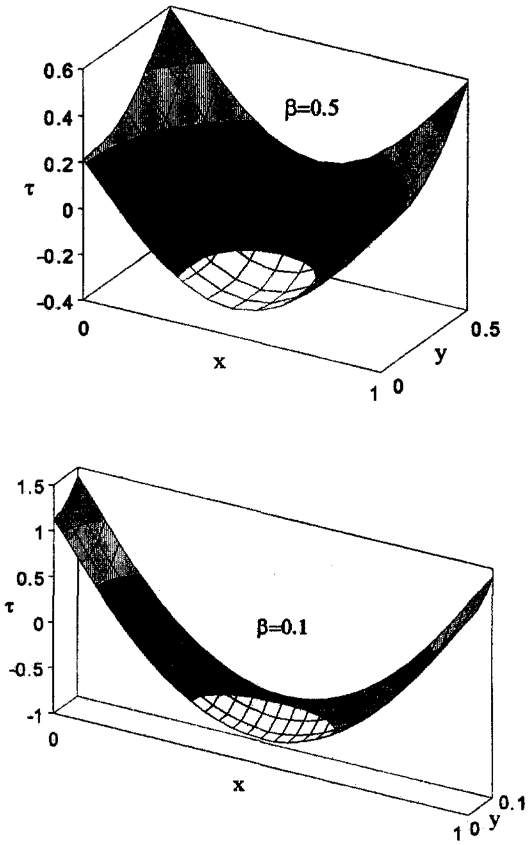

Fig. 3. Dimensionless temperature distribution, 3S version.

4), with $\beta = 0.1$, 0.5 and 1. The negative values of temperature are due to definition of τ; owing to the indetermination of B_{00}, the two-dimensional temperature profile can be shifted arbitrarily as one likes.

From Fig. 1, it comes out that the minimum and maximum fluid temperatures occur at the center and on the corners, respectively, of the rectangular cross section. For decreasing values of the aspect ratio, the minimum wall temperature (in the middle point of the long sides) approaches the minimum fluid temperature.

In Fig. 2, for the 3L version, the minimum fluid temperature lies on the straight line $y = \beta/2$, in proximity to the adiabatic wall. It is interesting to point out that the minimum fluid temperature is reached in $x = 0.292$ for any value of the aspect ratio (in the range 0.1–1), and that the fluid temperature has nearly the same value for any β, in the point $x = 0.6333$, $y = \beta/2$. The maximum values of fluid temperature are situated in the corners of the short heated wall. The solution to the 3S version is shown in Fig. 3, for $\beta = 0.1$ and 0.5; obviously for $\beta = 1$ (square duct) there is no difference between the versions 3L and 3S; the same holds for the versions 2L–2S and 1L–1S. The results put in evidence that the fluid temperature decreases starting from the long heated side, then reaches its minimum value, on $x = 1/2$, in proximity to the long adiabatic wall, remaining nearly constant up to this wall. This minimum position gets near the adiabatic wall for decreasing values of β.

Figure 4 refers to the 2L version; the wall temperature on the long sides becomes flatter for increasing aspect ratios, while the temperature variation along the short sides is very considerable; the minimum and maximum fluid temperatures occur in the center and on the corners, respectively, of the rectangle. The minimum wall temperature occurs in the middle point of the long heated sides for $\beta < 0.3687$, of the short adiabatic sides for $\beta > 0.3687$.

The version 2S is dealt with in Fig. 5; minimum

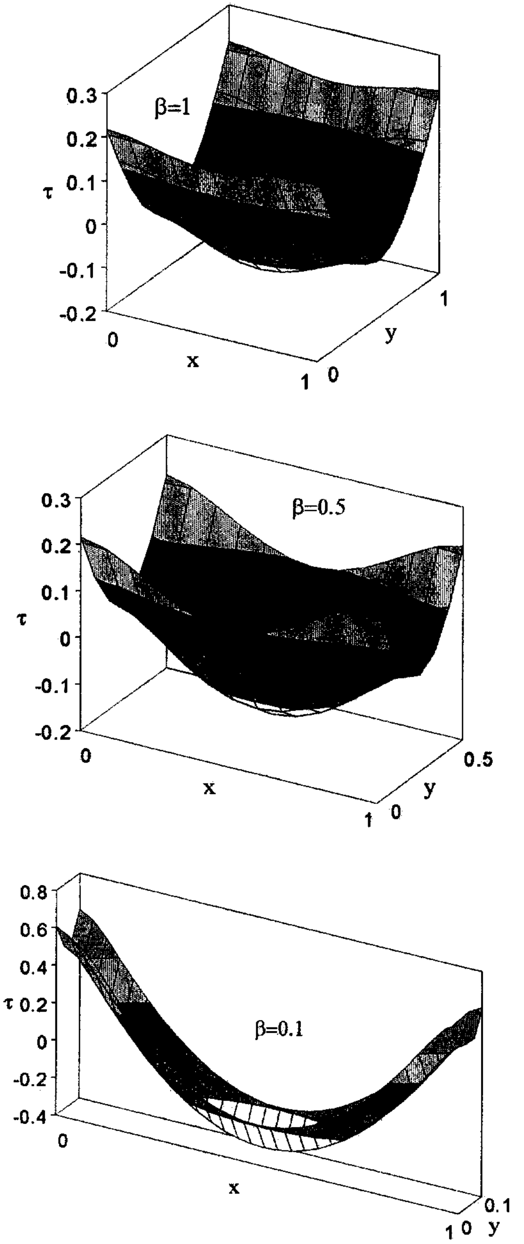

Fig. 4. Dimensionless temperature distribution, 2L version.

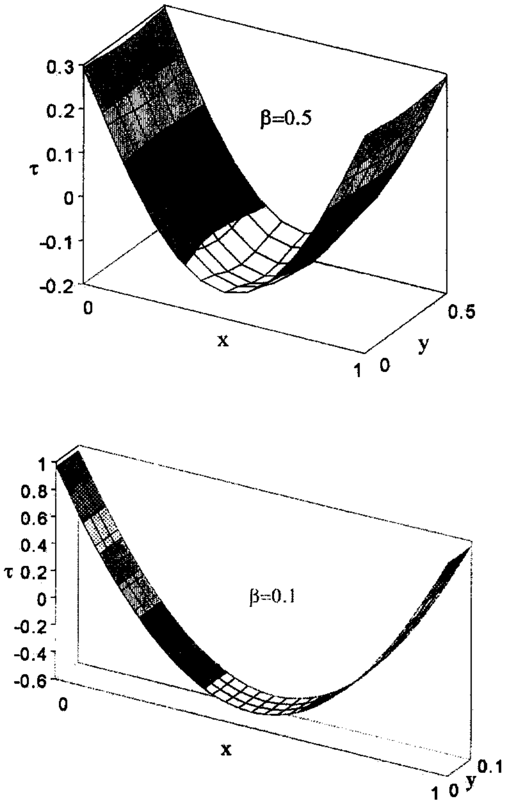

Fig. 5. Dimensionless temperature distribution, 2S version.

and maximum values of fluid temperature are situated at the center and on the corners, respectively. The wall temperature variation is negligible on the short heated sides, but very significant on the long adiabatic sides. The corner version (2C) is sketched in Fig. 6; the minimum and maximum wall temperatures are reached at the two corners between the adiabatic sides and the heated sides, respectively. The minimum fluid temperature occurs on the long adiabatic wall of the rectangle, in proximity to the adiabatic corner.

In Figs. 7 and 8 the 1L and 1S versions are presented; the fluid temperature is maximum on the tips of the heated side, minimum in the middle point of the opposite adiabatic side. For the 1S version, the wall temperature along the heated side is almost constant.

The temperature distributions are useful to determine Nu by means of equation (11); the Nusselt numbers are of paramount importance, because they are the fundamental tools for engineering applications and because they can be compared with the numerical results obtained by several authors for the four heated walls. Only this version has been extensively investigated in literature; Cheng [7] arrived at closed-form expressions for temperature and Nu, in the form of infinite series using separation of variables. Sparrow and Siegel [8] approached the same problem by a variational method and pointed out an error in [7] for the square duct, confirmed by Cheng [9], Shah [6] and Iqbal [10], while numerical lack of accuracy is found in [11] and [12].

For the version 4, the Nusselt numbers of the present paper are compared with the results of other authors. Table 2, where three-digit values are given, shows that Nu is nearly constant, decreasing 6% for β ranging from 1 to 0.1. For the square duct, the present value $Nu = 3.091$ is in perfect agreement with the results given in [6], [8], [9] and [10], while Chandrupatla *et al.* [11] and Lyczkowski *et al.* [12] proposed $Nu = 3.095$ and $Nu = 3.23$, respectively. To stress the fundamental role played by the velocity distribution on the thermal behavior, it is pointed out that, for slug flow, Gao and Hartnett [4] found

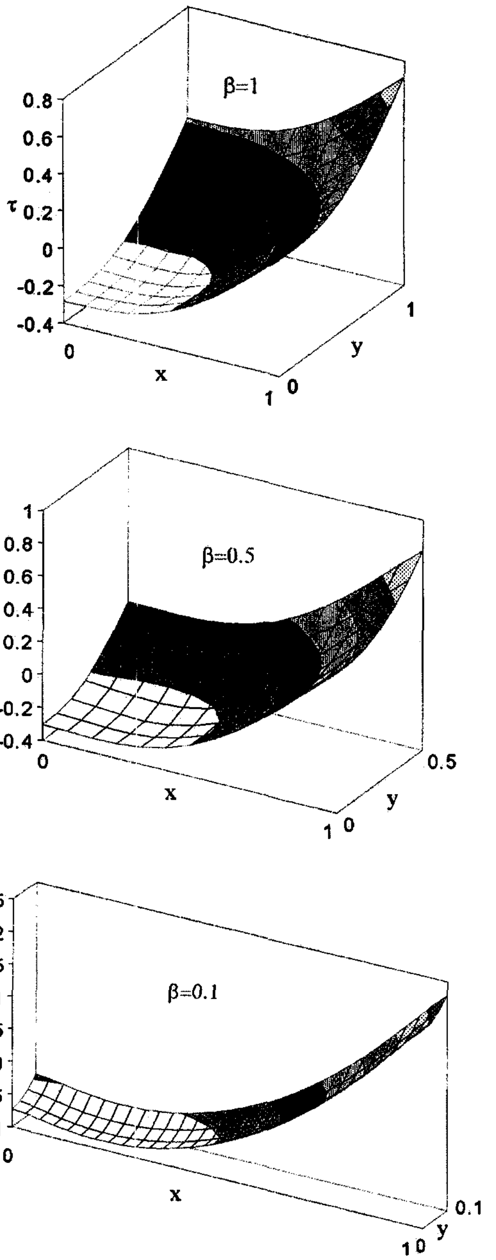

Fig. 6. Dimensionless temperature distribution, 2C version.

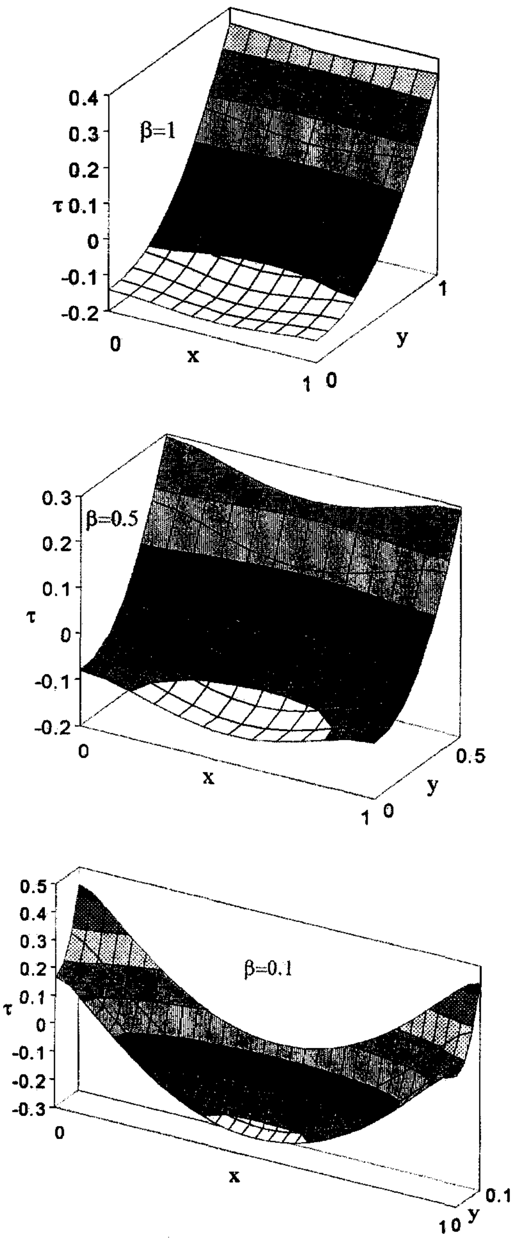

Fig. 7. Dimensionless temperature distribution, 1L version.

$Nu^* = 6$ for any β (Nu^* is nearly twice Nu). In order to verify the accuracy of the present solution, the temperatures defined in equations (13) and (14) have been quoted in Table 2, and compared with the analogous results of Table 6 in [6], where two-digit values are indicated. The agreement is very satisfactory. It should be pointed out that the Nusselt numbers presented in [2] for $\beta = 0.125$ and $\beta = 0.1$ (2.94 and 2.95, respectively), are probably printer's errors, as suggested by the value $Nu = 2.904$ for $\beta = 0.125$ reported in the preceding paper [13]. For $\beta = 0$, Nu can not reach the value 8.235 (the well known Nusselt number for slab geometry). As discussed in [2] and [8] this is justified by the fact that it is not possible to approach the slab geometry as a limiting case of four heated walls. It is a more rational idea to approach the slab geometry as a limiting case of the 2L version; the mathematical demonstration that equation (8), for the 2L version with $\beta \to 0$, coincides with the slab temperature profile will be object of future work. In Table 3, the Nusselt numbers, for all the remaining H2 versions, are presented, for different values of the aspect ratio. In literature there are no results about this matter (only in [4], but for slug flow, and in [14] the 2L and 2S versions are studied, with sinusoidal or parabolic heat flux distributions on the opposite heated walls), hence it is not possible to propose a comparison with other data. The highest values for Nu are observed for the 2L version. For the versions

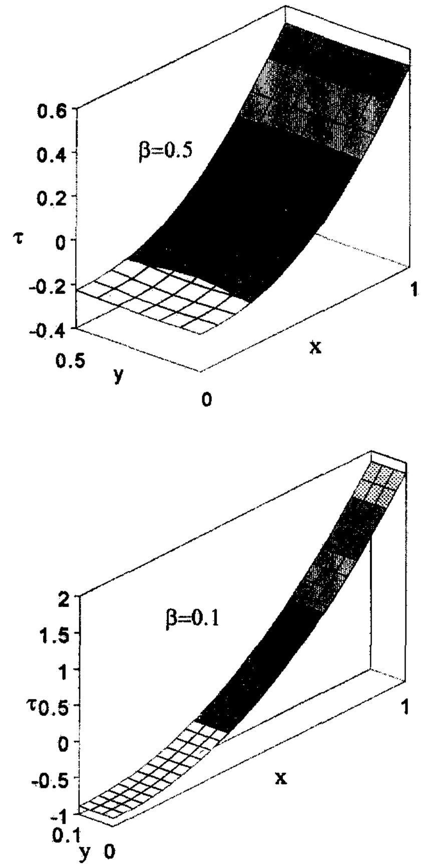

Fig. 8. Dimensionless temperature distribution, 1S version.

3S, 2S and 1S the Nusselt numbers increase with β, and the opposite happens for the versions 3L, 2L and 1L. In the case 2C (corner version) the Nusselt number is nearly constant (about 2.4, rather similar to $Nu^* = 3$, for any β, in [4]), increasing 2.5% for β ranging from 0.1 to 1.

For the sake of completeness, the minimum and maximum temperatures defined in equation (13) and (14) are reported in Tables 4 and 5.

Finally, a 5th order polynomial approximation is given, aimed at offering a very simple but accurate tool for technicians and designers involved in heat transfer applications:

$$Nu = \sum_{i=0}^{5} g_i \beta^i. \quad (15)$$

The coefficients g_i are given in Table 6 for all the eight versions of H2 conditions considered; the relative difference ε is positive when the equation (15) gives Nusselt numbers greater than the rigorous calculations of equation (11), it is negative otherwise. This tool is very significant, because the agreement between Nu from the approximate equation (15) and the exact equation (11) is always within -0.063% and $+0.025\%$.

CONCLUDING REMARKS

This paper has reported a study on Nusselt number predictions for laminar flow in rectangular ducts, for the eight versions of the H2 boundary conditions. The mathematical procedure, based on the double Fourier transform, is not valid for the slab geometry $\beta = 0$. The temperature distributions are obtained, apart from an additive constant, as a double series of cosinusoidal functions, with coefficients depending on the different combinations of heated and adiabatic walls. By integrating temperature differences only, the Nusselt numbers are accurately predicted and compared with the results obtained numerically by several authors, mainly for the four heated walls. At last the Nusselt numbers for all the eight versions of the H2 boundary conditions are presented in terms of 5th order polynomials (with the absolute value of the relative error ε less than 0.063%), offering an exhaustive solution to the various versions of the H2 problem.

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Table 2. Nusselt number, maximum and minimum wall temperature; four heated walls

β	Nu	Nu [2]–[6]	Nu [8]	t_{max}	t_{max} [6]	t_{min}	t_{min} [6]
1	3.091	3.091	3.09	1.399	1.39	0.769	0.769
5/6	3.085	—	—	1.407	—	0.691	—
0.75	3.077	3.07	—	1.418	1.41	0.648	0.649
2/3	3.064	—	—	1.436	—	0.602	—
0.5	3.022	3.02	—	1.503	1.5	0.499	0.499
1/3	2.964	2.97	—	1.644	—	0.379	—
0.25	2.935	2.94	—	1.770	1.76	0.311	0.311
0.2	2.922	2.93	—	1.878	—	0.266	—
0.125	2.909	—	—	2.115	2.11	0.192	0.192
0.1	2.907	—	2.90	2.226	—	0.163	—

Table 3. Nusselt numbers, for different aspect ratios, in H2 conditions

β	3L	3S	2L	Nu 2S	2C	1L	1S
1	2.943	2.943	4.083	4.083	2.430	2.686	2.686
5/6	3.009	2.869	4.376	3.780	2.428	2.913	2.457
0.75	3.042	2.824	4.532	3.603	2.425	3.041	2.324
2/3	3.076	2.773	4.700	3.404	2.421	3.179	2.178
0.5	3.140	2.648	5.058	2.932	2.408	3.494	1.834
1/3	3.204	2.494	5.445	2.321	2.389	3.870	1.399
0.25	3.241	2.408	5.654	1.934	2.380	4.089	1.132
0.2	3.268	2.354	5.790	1.662	2.376	4.233	0.952
0.125	3.319	2.270	6.014	1.172	2.371	4.471	0.645
0.1	3.339	2.240	6.096	0.981	2.371	4.558	0.531

Table 4. Maximum temperature t_{max}, for different aspect ratios, in H2 conditions

β	3L	3S	2L	t_{max} 2S	2C	1L	1S
1	1.636	1.636	1.058	1.058	1.696	1.023	1.023
5/6	1.709	1.581	1.085	1.038	1.707	1.035	1.015
0.75	1.761	1.558	1.105	1.030	1.723	1.044	1.012
2/3	1.826	1.540	1.132	1.023	1.751	1.056	1.009
0.5	2.019	1.524	1.218	1.012	1.856	1.097	1.005
1/3	2.366	1.567	1.383	1.005	2.100	1.185	1.002
0.25	2.658	1.636	1.525	1.003	2.341	1.270	1.001
0.2	2.904	1.708	1.647	1.002	2.564	1.349	1.001
0.125	3.453	1.901	1.917	1.001	3.128	1.552	1.000
0.1	3.713	2.004	2.046	1.001	3.428	1.662	1.000

Table 5. Minimum temperature t_{min}, for different aspect ratios, in H2 conditions

β	3L	3S	2L	t_{min} 2S	2C	1L	1S
1	−0.0323	−0.0323	0.2143	0.2143	−0.2756	−0.2108	−0.2108
5/6	−0.0196	−0.0428	0.2796	0.1611	−0.2760	−0.2035	−0.2188
0.75	−0.0116	−0.0478	0.3231	0.1354	−0.2769	−0.1997	−0.2237
2/3	−0.0007	−0.0527	0.3771	0.1106	−0.2792	−0.1958	−0.2293
0.5	0.0295	−0.0612	0.5318	0.0653	−0.2905	−0.1879	−0.2433
1/3	0.0789	−0.0654	0.6970	0.0293	−0.3247	−0.1786	−0.2632
0.25	0.1142	−0.0636	0.6107	0.0163	−0.3644	−0.1712	−0.2765
0.2	0.0849	−0.0603	0.5464	0.0103	−0.4031	−0.1642	−0.2858
0.125	−0.0655	−0.0503	0.4212	0.0040	−0.5043	−0.1453	−0.3018
0.1	−0.1311	−0.0450	0.3671	0.0025	−0.5580	−0.1345	−0.3076

Table 6. Polynomial coefficients appearing in equation (15)

Version	g_0	g_1	g_2	g_3	g_4	g_5	ε [%]
4	2.9235	−0.3737	2.4553	−3.5494	2.0489	−0.4135	−0.023
3L	3.4487	−1.3586	3.0661	−4.5527	3.2042	−0.8646	−0.013
3S	2.1197	1.2439	−0.3927	0.2615	−0.6206	0.3312	+0.025
2L	6.4812	−4.4032	6.4748	−10.3513	8.6349	−2.7534	−0.015
2S	0.0470	11.0106	−18.9052	24.3137	−17.5238	5.1407	−0.063
2C	2.3766	−0.1309	0.8612	−1.3250	0.8534	−0.2053	+0.018
1L	4.9460	−4.2699	4.2859	−4.3557	2.9119	−0.8321	−0.010
1S	0.0094	5.8307	−6.7178	6.3174	−3.6781	0.9245	−0.047

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